## Fall 2013 Interstellar Problems

- 21. How many positive integers less than 50 have an odd number of positive integer divisors?
- 22. Let f be the function defined by  $f(x) = ax^2 \sqrt{2}$  for some positive a. If  $f(f(\sqrt{2})) = -\sqrt{2}$  then a =

(A) 
$$\frac{2-\sqrt{2}}{2}$$
 (B)  $\frac{1}{2}$  (C)  $2-\sqrt{2}$  (D)  $\frac{\sqrt{2}}{2}$  (E)  $\frac{2+\sqrt{2}}{2}$ 

- 23. What is the sum of the angle measures of all the angles on all seven faces of a right prism whose base is a regular pentagon?
- 24. An acute isosceles triangle, ABC, is inscribed in a circle. Through B and C, tangents to the circle are drawn, meeting at point D. If  $\angle ABC = \angle ACB = 2\angle D$  and x is the radian measure of  $\angle A$ , then x =



(A) 
$$\frac{3}{7}\pi$$
 (B)  $\frac{4}{9}\pi$  (C)  $\frac{5}{11}\pi$  (D)  $\frac{6}{13}\pi$  (E)  $\frac{7}{15}\pi$ 

- 25. Four whole numbers, when added three at a time, give the sums 180, 197, 208, and 222. What is the largest of the four numbers?
- 26. At one of George Washington's parties, each man shook hands with everyone except his spouse, and no handshakes took place between women. If 13 married couples attended, how many handshakes were there among these 26 people?1
- 27. How many of the numbers, 100, 101, ..., 999, have different digits in increasing order or in decreasing order?
- 28. First a is chosen at random from the set  $\{1, 2, 3, \ldots, 99, 100\}$ , and then b is chosen at random from the same set. The probability that the integers  $3^a + 7^b$  has units digit 8 is

(A) 
$$\frac{1}{16}$$
 (B)  $\frac{1}{8}$  (C)  $\frac{3}{16}$  (D)  $\frac{1}{5}$  (E)  $\frac{1}{4}$ 

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- 29. For how many integers N between 1 and 1990 is the improper fraction  $\frac{N^2+7}{N+4}$  not in lowest terms?
- 30. In the figure, ABCD is a quadrilateral with right angles at A and C. Points E and F are on  $\overline{AC}$ , and  $\overline{DE}$  and  $\overline{BF}$  are perpendicular to  $\overline{AC}$ . If AE = 3, DE = 5, and CE = 7, then BF =



(A) 3.6 (B) 4 (C) 4.2 (D) 4.5 (E) 5